

Quantitative Finance and Investment Portfolio Management Exam

# **Exam QFIPM**

Date: Friday, April 28, 2023

#### **INSTRUCTIONS TO CANDIDATES**

#### **General Instructions**

1. This examination has 13 questions numbered 1 through 13 with a total of 70 points.

The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

#### Written-Answer Instructions

- 1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.
  - a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example,  $\beta_1$  can be typed as beta\_1 (and ^ used to indicate a superscript).
  - b) In the Excel document formulas should be entered. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit.
  - c) Individual exams may provide additional directions that apply throughout the exam or to individual items.
- 2. The answer should be confined to the question as set.
- 3. Prior to uploading your Word and Excel files, each file should be saved and renamed with your five-digit candidate number in the filename.

4. The Word and Excel files that contain your answers must be uploaded before time expires.

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#### **Navigation Instructions**

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



(5 *points*) ABC company offers an U.S. 401(k) plan to its employees. The company recently added four indexed bond funds, named A, B, C, and D as shown below, to its 401(k) investment options. These four indexed funds are managed by XYZ Associates, an investment management firm.

Fund	Tracking Index	Duration	Investment Grade?
А	Barclay's Corporate High-Yield Bond Index	Medium	Below
В	Barclay's Corporate Intermediate Bond Index	Medium	Yes
С	Merrill Lynch 1 to 3 Year Corporate Bond	Short	Yes
	Index		
D	Merrill Lynch Long-term Corporate Bond Index	Long	Yes

Assume that people are more risk averse the older they become.

(a) (*1 point*) Assess which index fund (or funds) from the above table is most suitable for each of the three types of employees: "young", "mid-aged", and "near retirement".

ANSWER:			

The tracking error for one of the funds managed by XYZ Associates over the last 10 years is 100bp, as shown below:

Year	1	2	3	4	5	6	7	8	9	10
Fund Return	12.0%	7.0%	Х	-3.0%	4.0%	3.3%	5.4%	3.8%	5.0%	3.7%
Index Return	12.6%	6.5%	1.2%	-5.0%	4.1%	3.2%	5.1%	5.7%	4.6%	3.8%
Note: Tracking e	rror over the	10-year p	eriod = 10	0bp						

(b) (1.5 points) Explain how six different approaches could be used to reduce the tracking error.

XYZ Associates created a five-bond portfolio as below that meets its client's "target dollar duration" requirement. This portfolio is to be managed to maintain the "target dollar duration".

Portfolio	Portfolio at the inception				Portfolio at some time t after the inception		
	Market Value	<b>Duration</b>			Market Value	Duration	
Bond1	\$10,800,000	1.25		Bond1	\$10,260,000	0.60	
Bond2	\$10,300,000	4.45		Bond2	\$10,403,000	4.10	
Bond3	\$10,600,000	5.05		Bond3	\$10,812,000	4.55	
Bond4	\$10,000,000	5.65		Bond4	\$10,050,000	5.05	
Bond5	\$10,100,000	5.95		Bond5	\$10,201,000	5.55	

(c) (1.5 points) Propose two different trades (without using derivatives) that could be done at time t to meet the target dollar duration requirement.

The response for this part is to be provided in the Excel document

A proposal is made to trade U.S. treasury futures contracts to maintain the target dollar duration. At time t, the price of the cheapest-to-deliver bond is \$100,000, the conversion factor = 1.16 and the number of contracts to be traded to maintain the target dollar duration is 6.

(d) (*1 point*) Calculate the duration of the cheapest-to-deliver bond, indicating if the futures trade is a "buy" or "sell".

(6 points) You are an investment actuary on the ALM team of ABC Life. You are considering the following fixed income securities (all rates are annually compounded, and coupons are paid annually):

Bond	Maturity	Coupon (%)	Price	Yield (%)	DV01	Mod Dur	Mac Dur
Α	3 yr	3		3.5	A1	A2	
В	10 yr	4	92.28	5	7.71	B1	B2
С	10 yr	0				C1	

(a) (1.5 points) Calculate the values A1, A2, and B2 in the table.

ANSWER:			

(b) (1.5 points) Describe the properties of B1 and C1 in the table, considering how bond durations are generally impacted by the coupon rates.

ANSWER:

The valuation team has performed sensitivity runs for ABC Life's insurance liabilities and communicated that because policyholder behavior is sensitive to market conditions, dynamic lapse assumptions needed to be used in the modeling.

- (c) (*1 point*)
  - (i) Explain the relationship between the convexity of Bond B and the convexity of Bond C at issue.

#### ANSWER:

(ii) Describe the considerations for selecting a bond to match the insurance liabilities.

ABC's management is concerned with the risks of large movements in the yield curve and the potential impact on their annuity products. Your colleague proposes using a position of k units of bond B to dollar duration-match a block of annuity products that has a fair value of 120 million and a modified duration of 8 years.

(d) (*1 point*) Calculate the position k.

ANSWER:

(e) (*1 point*) Describe a key limitation of your colleague's method.

(6 points) Company A, a publicly traded copper mining company has just issued a oneyear, zero-coupon bond with face amount equal to 100 million. The fair value of the firm's assets is evaluated at 115 million.

You are given the following additional information:

- The company currently does not have other debts
- The company does not pay any dividends
- The volatility  $(\sigma_A)$  of the value of the company's assets is 10%
- The expected growth rate  $(\mu_A)$  of the value of the company's assets is 5%
- The continuously compounded risk-free rate is 2%

You are using Merton's model.

- (a) (2 points) Calculate the following for Company A over a one-year horizon:
  - (i) The real-world probability of default

#### ANSWER:

(ii) The risk-neutral probability of default

ANSWER:

The value of the zero-coupon bond from your model is 97.78.

(b) (*1 point*) Calculate the risk-neutral expected rate of recovery, conditional on default of the bond.

ANSWER:

(c) (*1 point*) Describe the two general model types into which credit risk models are often divided.

ANSWER:

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Bank C specializes in making loans to mining companies. Bank C also owns the zerocoupon bond issued by Company A. The risk team at Bank C uses the single factor Gaussian copula model for calculating capital charges for its portfolio.

(d) (2 *points*) Critique the risk team's choice of this model, including its advantages and disadvantages.

(6 points) You recently joined Company ABC in its Investment Risk Department. ABC is an energy-related technology company. One of your mandates is to develop an Investment Policy Statement (IPS) for the company's defined benefit (DB) pension plan.

You are doing some research first before developing the IPS. You plan to calculate the plan funding ratio (with future compensation increases reflected) and understand the split between retired-lives and active-lives.

(a) (*1 point*) Recommend the use of accumulated benefit obligation (ABO) or projected benefit obligation (PBO) to calculate the funding ratio.

ANSWER:

(b) (*1 point*) Explain why the split between retired-lives and active-lives is important when assessing the funding status of the DB pension plan.

ANSWER:

You gather the following information from your research work:

- ABC is a small company.
- Most pension plan members are either active-lives and older than age 50, or are retired.
- Plan asset value: \$300 million, ABO: \$350 million, and PBO: \$400 million.
- The company has rising earnings and a strong (low debt) balance sheet.
- The plan's portfolio is actively invested in energy sector.
- (c) (2 *points*) Describe the implications of the above information for the plan's risk objective.

The IPS has been developed and the return objective has been set to 6% (equal to the discount rate used to determine the PV of liabilities). After several years of operating, the plan now has a funding ratio of 110%. Also, the Company expanded and hired a significant number of young employees and is still growing.

(d) (*1 point*) Recommend an updated return objective.

# ANSWER:

From the corporate risk management perspective, senior management has the following concerns:

- How to better manage pension investments in relation to operating activities?
- How to better coordinate pension investments with pension liabilities?
- (e) (1 point) Describe strategies that could address management's concerns.

When necessary, reference the formula in the formula sheet. You do not need to write down the explicit formula in your answer.

# 5.

(6 points)

You are a fund manager at a large defined benefit pension plan. You are given the following statistics on two portfolios that could be used to back your pension liability:

Year	Portfolio 1	Portfolio 2	Liability	Return	Return
				(Portfolio 1)	(Portfolio 2)
2016	101	101	100		
2017	103	103	102	1.98%	1.98%
2018	104	105	104	0.97%	1.94%
2019	109	108	106	4.81%	2.86%
2020	110	109	108	0.92%	0.93%
2021	114	112	110	3.64%	2.75%
Mean return				2.46%	2.09%
Standard					
Deviation				1.71%	0.78%

The risk-free interest rate is assumed to be 1.5%.

(a) (*1 point*) Justify, based on the Sharpe Ratio, which portfolio is preferable in the asset-only asset allocation strategy?

#### ANSWER:

(b) (*1 point*) Describe the shortcomings of the Sharpe ratio in measuring the trade-off between risk and return in an asset-liability framework.

ANSWER:

(c) (2 *points*) Recommend which portfolio should be selected in the asset-liability management framework based on the Risk Adjusted Change in Surplus (RACS).

Upon revaluation at the end of 2021, the pension plan has \$115 million of assets compared to a liability of \$110 million. You are diversifying the investments between the portfolio selected in part (c) and a bond index fund with the following characteristics:

			Correlation	
Mean return	1.8%	Portfolio 1	Portfolio 2	Bond Index
				Fund
Standard	0.5%	1		
Deviation				
Duration	10	0.06	1	
(Bond Index				
Fund)				
Duration	14	0.12	0.15	1
(Liability)				

The duration-matching relation between the liability return  $(R_{L,t})$  and the bond index fund return  $(R_{B,t})$  is given as follows:

$$R_{L,t} - R_f = \beta \big( R_{B,t} - R_f \big) + \varepsilon_t,$$

with  $E[\varepsilon_t] = 0$ .

As the first step, you want to know the minimum percentage asset allocation to the portfolio selected in part (c) needed for your fund to prevent the surplus from shrinking. An analyst on your team did the calculation, and concluded that the minimum percentage was 3.04%. You checked their calculation and found that they have used a wrong mean portfolio return of 3%.

(d) (1.5 points) Calculate the correct minimum percentage asset allocation to the portfolio selected in part (c) based on the correct mean portfolio return.

ANSWER:

(e) (0.5 points) Explain how the minimum percentage in part (d) changes if the liability value is unchanged but the asset value is \$90.

(5 points) You are a consultant who has been hired by XYZ Assurance, a small life insurance company, to provide investment expertise. XYZ primarily offers 10 year term insurance, but has recently started to write universal life products. XYZ is hoping that offering permanent products such as universal life may provide an opportunity to improve their portfolio yield.

At your first meeting, XYZ's investment team shares ideas among themselves. A junior member suggests purchasing mortgage debt, and your boss asks for your opinion.

(a) (*1 point*) Describe three differences between residential and commercial mortgages.

ANSWER:

(b) (0.5 points) Identify four typical mortgage covenants.

ANSWER:

ANSWER:

(c) (0.5 points) Describe the borrower's put option in a non-recourse mortgage.

Your boss feels uncertain about purchasing mortgage debt outright, citing concerns about concentration and default risk given XYZ's relatively small asset portfolio.

- (d) (*1 point*) Describe the securitization process of commercial mortgage-backed securities, specifically identifying how the following roles are involved:
  - Investment Bankers
  - Rating Agencies
  - Investors
  - Servicers

After some discussion and research, XYZ's team decides to invest into a CMBS offered by ABC Investments. This CMBS has a \$100 million pool comprised of ten \$10 million dollar mortgages with no prepayment penalties. Three tranches are available:

- Tranche A: Senior/Investment Grade
- Tranche B: Junior/Non-Investment Grade
- Tranche X: Senior Interest Only (IO)

Your boss suggests purchasing a position in Tranche X to back the liabilities of the company.

(e) (1 point) Explain why you should not invest in Tranche X

#### ANSWER:

(f) (*1 point*) Recommend an alternative tranche.

# **7.** (7 points)

(a) (2 *points*) Contrast the valuations of REIT and direct investing from both the micro and the macro perspectives.

ANSWER:

(b) (0.5 points) Explain the challenges related to behavioral finance in real estate investment when compared to equity market investment.

ANSWER:

Consider the following behavioral biases:

- (i) Framing bias
- (ii) Home bias
- (iii) Herding bias
- (c) (3 points)
  - (i) (1.5 points) Explain how each bias affects real estate investing.

ANSWER:

(ii) (1.5 points) Describe workarounds for each bias.

ANSWER:

Your colleague made the following statements:

Statement A: "REIT A has an average cost of capital of 10%, REIT B has an average cost of capital of 12%. This implies REIT A can afford to pay more for property X than REIT B."

Statement B: "Income contributes the bulk of the total returns on real estate investment, as well as most of the volatility."

(d) (1.5 points) Critique your colleague's statements.

(4 *points*) You were asked to evaluate the performance of your company's investment portfolio through performance attribution.

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(a)	(I)	boint)	Describe	macro	and	micro	performance	attribution.

ANSWER:			

You were given a draft micro attribution analysis using a fundamental factor model:

(%)	Portfolio	Normal	Factor Return	Return
	Exposure	Exposure	(Beta)	
Market				5.05
Normal Portfolio				4.95
Cash Timing	2.05	0.05	-0.05	
Beta Timing	1.04	1.00	2.00	
Total Market				
Timing				
Growth	1.23	0.51	-0.55	
Size	-0.50	0.86	0.57	
Yield	-0.90	-0.54	2.44	
Total				
Fundamental				
<b>Risk Factors</b>				
Basic Industry	35.23	41.00	-0.04	
Financials	40.12	37.00	0.01	
Consumer	24.65	22.00	0.02	
Total				
Economic				
Sectors				

The zero factors are 0 for all fundamental factors. The actual portfolio return is 5.5%.

- (b) (*1 point*)
  - (i) Define Normal Exposure

#### ANSWER:

(ii) Explain the differences between normal exposure and portfolio exposure in the chart above.

ANSWER:

(c) (1.5 points) Calculate the attribution of the active impact into market timing, fundamental risk factors, and economic sectors.

ANSWER:

(d) (0.5 points) Calculate the portfolio return predicted by this model.

(5 *points*) ABC Life is an insurance company that sells a mixture of insurance products including Term and Whole Life, Immediate and Deferred Annuities, and Universal Life. You joined ABC Life as its first actuary on the credit portfolio management team.

ABC Life's approach for supporting its liabilities consists of the following elements:

- ABC Life entered a reinsurance treaty with a reputable AA rated reinsurer that covers any excess mortality loss over 100% of the latest industry mortality table for life insurance.
- ABC Life entered an interest rate swap agreement with FGH Bank as follows:
  - Notional amount of \$100M.
  - FGH Bank pays ABC Life a fixed rate of 1%.
  - ABC Life pays FGH Bank the one-year point on the US Treasury par yield curve.
  - The term of the swap is 5 years.
- ABC Life underwrites \$100M in commercial mortgage loans, with the underlying real estate as collateral.
- ABC Life invests \$100M in government and municipal bonds from emerging markets such as Brazil, Russia, India, and China.
- ABC Life invests \$100M in funds tracking the S&P 500 index.
- (a) (2 points)
  - (i) Identify the source of credit risk, if any, inherent in each element of ABC Life's current approach.

#### ANSWER:

(ii) Assess qualitatively the degree of credit risk in each element.

ABC Life just started monitoring credit risks exposure. You consider the team to be a Level 1 credit portfolio manager, as described in the Handbook of Credit Risk Management.

(b) (1.5 points) Describe three key functions of the credit portfolio management team.

ANSWER:			

You have been given the goal of improving the credit portfolio management team to Level 2, as described in the Handbook of Credit Risk Management.

(c) (1.5 points) Explain three additional key functions that need to be implemented in order to achieve this goal and the importance of each function.

(5 *points*) You are a portfolio manager for a long-term investor who is considering purchasing 20-year US-Treasury notes with a par value of \$5 million and a coupon of 7% paid annually. The prevailing market interest rate is 3%. You would like to fund a position in three days using these securities as collateral.

You contract with manager Y to borrow the funds needed.

(a) (1.5 points) Describe how you would construct a repurchase agreement.

ANSWER:

You contract with manager Z to borrow the funds needed.

(b) (1.5 points) Describe how you would construct a securities lending agreement.

ANSWER:

Suppose you instead purchase Treasury Inflation-Protected Securities (TIPS) bonds with a 20-year maturity with a par value of \$5 million, a TIPS real yield of 3%, risk adjusted break-even inflation rate of 3.1%, and nominal yield of 7%.

(c) (*1 point*) Calculate the risk premium embedded in the bonds.

ANSWER:

(d) (*1 point*) Explain why the portfolio manager would consider the TIPS bonds over the US Treasury notes.

(5 *points*) CDF Fund, an education endowment fund of \$150 million that supports and funds teaching and research for ABC university is due for annual asset allocation review. The treasurer has prepared some capital information that will facilitate adjustment of the current year asset allocation.

ABC university has a return requirement of 6.77%. ABC does not want to borrow to purchase risky assets. No substantive relevant legal and regulatory factors or unique circumstances affect the asset allocation.

The risk objective is to accept lowest possible standard deviation of return and not greater than 8.0% while maximizing Sharpe Ratio.

	Return	Std Dev	US Equities	Asia Equities	Intermediate Bonds	Long Term Bonds	International Bonds	Real Estate	Sharpe ratio
1 US Equi	ties 12.00%	22.00%	1	0.54	0.35	0.5	0.24	0.32	0.4545
2 Asia Equ	ities 9.50%	18.00%	0.54	1	0.04	0.28	0.36	0.13	0.4167
3 Intermed	iate								
Bonds	4.25%	6.00%	0.35	0.04	1	0.85	0.48	0.11	0.3750
4 Long Ter	m								
Bonds	5.25%	8.00%	0.5	0.28	0.85	1	0.38	-0.15	0.4063
5 Internatio	onal								
Bonds	5.60%	9.50%	0.24	0.36	0.48	0.38	1	0.12	0.3789
6 Real Esta	ate 6.50%	13.00%	0.32	0.13	0.11	-0.15	0.12	1	0.3462

The risk-free rate is 2%

8 corner portfolios have been provided which define the minimum-variance frontier.

Corner Portfolio Number	1	2	3	4	5	6	Return	Std Dev	Sharp Ratio
1	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12.00%	22.00%	45.45%
2	68.00%	32.00%	0.00%	0.00%	0.00%	0.00%	11.20%	18.71%	49.17%
3	49.57%	32.45%	0.00%	0.00%	0.00%	17.98%	10.20%	15.72%	52.16%
4	34.63%	25.14%	0.00%	0.00%	17.67%	22.56%	9.00%	12.56%	55.74%
5	0.00%	10.63%	0.00%	47.14%	14.41%	27.81%	6.10%	6.45%	63.58%
6	0.00%	10.36%	69.49%	0.00%	5.24%	14.91%	5.20%	5.58%	57.38%
7	0.00%	0.00%	93.33%	0.00%	0.00%	6.67%	4.40%	5.76%	41.67%
8	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	4.25%	6.00%	37.50%

(a) (2 *points*) Recommend a strategic asset allocation (asset class weights) for the CDF Fund that satisfies the return requirements and risk objectives.

ANSWER:

ANSWER:

The treasurer would prefer not to invest in real estate because real estate is highly illiquid, its return objective is below the target, has a high standard deviation above the target, and the Sharpe Ratio is low.

(b) (1 point) Explain why real estate should be included in the current portfolio.

Assume the fund changes its management strategy to maximize the mean-variance investor's utility  $U_m = E(R_m) - 0.50R_A\sigma_m^2$ , where expected return and standard deviation are in decimal form and 0.5 is a scaling factor.

Suppose the fund decides to restrict its choice in investing to portfolios 3 and 4.

(c) (*1 point*) Calculate the values of the parameter  $R_A$  for which the fund would prefer portfolio 4 to portfolio 3.

ANSWER:

(d) (*1 point*) Explain whether choosing portfolio 4 over portfolio 3 would be consistent with the expected risk aversion of an endowment fund.

(6 points)

(a) (0.5 point) Define leveraged buyouts (LBOs).

ANSWER:

An LBO firm offers \$900 million to purchase the equity of Company A and to pay off the outstanding debt. The \$900 million LBO is financed with \$800 million in debt and \$100 million in equity. By foregoing dividends and using the free cash flow to pay down the existing debt, the management of the company can own the company free and clear in nine years. Use a long-term growth rate of 4% per year and a discount rate of 12%. EBITDA represents the free cash flow from operations that is available to the owners and debtors of the company.

Company A	In \$ millions
Market value of Equity	600
Face value of Debt	200
EBITDA	180
EBITDA after LBO	200

(b) (0.5 point) Calculate the annual compounded return for this investment.

ANSWER:	

(c) (*1 point*) Identify four methods that the management of the company can use to realize the value of the LBO deal.

The LBO firm now considers two new buyout candidates, Company B and Company C. During last year, Company B shares declined approximately 17% and Company C shares move up approximately 2%.

Income Statement (In \$ thousands)	Company B	Company C	Balance Sheet (In \$ thousands)	Company Company B C			
Net sales	16,000	19,000	Current assets	6,000	5,000		
Less cost of goods sold	10,000	14,000	Noncurrent assets	3,500	6,000		
Less selling & admin.	4,000	3,000	Other assets	400	200		
Operation income	2,000	1,800	Total assets	9,900	11,200		
Interest expense	300	500	Current liabilities	2,000	5,000		
Nonoperating expenses	250	300	Noncurrent liabilities	3,000	3,500		
	1,450	1,000	Total debt	5,000	8,500		
Income tax	305	210					
Net income	1,145	790	Shareholders' equity	4,900	2,700		
Free Cash Flow (In \$ <u>thousands)</u> Free cash flow	thousands) Company B Company C						

(d) (2 points) Recommend an LBO candidate to the firm.

ANSWER:

(e) (*1 point*) Identify four benefits of LBOs to both corporate management and investors.

ANSWER:

(f) (*1 point*) Compare two private equity strategies: venture capital and leveraged buyouts.

(4 *points*) At the beginning of Year 1, a hedge fund manager purchases ten convertible bonds with the information below.

Convertible Bond par value	\$1,000
Convertible Bond coupon	6.00%
Convertible Bond market price	\$900
Current price of the underlying stock	\$40
Delta for the bond	0.4

(a) (*1 point*) Calculate the position and shares of underlying stock for the manager to hedge the equity exposure of the convertible bond.

ANSWER:

To make arbitrage profits, the manager goes on to have the equity exposure at the beginning of Year 1 as determined in part (a) above. Assume that:

- The manager receives a short rebate of 4.5%.
- The price of the stock increases to \$45 at the end of Year 1.
- The price of the convertible bond increases to \$950 at the end of Year 1.
- (b) (2 *points*) Calculate the profit for this arbitrage strategy over the holding period of Year 1.

ANSWER:

Assume that the manager uses a leverage ratio of 2:1 in the purchase of the convertible bonds and borrows the additional investment capital from his prime broker at a prime rate of 4%.

(c) (*1 point*) Calculate the total return on capital for this arbitrage.

ANSWER:

#### **\*\*END OF EXAMINATION\*\***