Long-Term Actuarial Mathematics

Exam LTAM

Date: Friday, April 24, 2020
Time: 8:30 a.m. – 12:45 p.m.

Recognized by the Canadian Institute of Actuaries.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____. Your name must not appear.

2. Do not break the seal of this book until the supervisor tells you to do so.

3. Tables for this examination will be distributed by the Supervisor.

4. This examination has a total of 96 points. It consists of:

   Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

   Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

   You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

   Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.

   No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:

   (a) Name
   (include last name, first name and middle initial)

   (b) Candidate Number
   (Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)

   (c) Test Site Code
   (The supervisor will supply the number.)

   (d) Examination Part
   (Code the examination that you are taking by blackening the circle to the left of "Exam LTAM")

   (e) Booklet Number
   (The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

   In the box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

   In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded**.

   Leave the boxes titled "Test Code" and "Form Code" blank.

   On the back of the answer sheet fill in the Booklet Number in the space provided.

CONTINUED ON INSIDE FRONT COVER

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Exam LTAM - Front Cover

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.

4. Five answer choices are given with each multiple-choice question, each answer choice being identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.

5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.

6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.

8. After the examination, the supervisor will collect this book and the answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE. All books and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

Written-Answer Instructions

1. **Write your candidate number at the top of each sheet. Your name must not appear.**

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert your written-answer sheets that you want graded into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam LTAM.

6. **Sign your essay answer envelope. If it is not signed, your examination will not be graded.**

7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

   In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

   For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam LTAM

SECTION A – Multiple-Choice

****BEGINNING OF EXAMINATION****

1. Which of the following is most likely to be true with respect to a structured settlement following a serious injury to the injured party?

(A) There will be no payment until the time of Maximum Medical Improvement.

(B) If the injured party does not recover sufficiently to return to work, then the annuity benefits will be reduced.

(C) Structured settlements decrease the injured party's dissipation risk compared with a lump sum benefit.

(D) The injured party is generally better off using a structured settlement buy-out from a specialist firm.

(E) The structured settlement value would be determined using unadjusted standard mortality tables.

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2. You are given the following extract from a two-year select mortality table:

x	$l_{[x]}$	$l_{[x-1]+1}$	l_x
80	40,695	41,988	42,445
81	36,398	37,700	38,162
82	32,109	33,402	33,862
83	27,888	29,154	29,605

You are also given that deaths are uniformly distributed between integer ages.

Calculate ${}_{0.8}q_{[80]+0.6}$.

(A) 0.058

(B) 0.062

(C) 0.066

(D) 0.070

(E) 0.074

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3. For a special fully discrete 3-year term insurance with level premiums on (x), you are given:

(i) The death benefits, b_{k+1}, and mortality rates are:

k	b_{k+1}	q_{x+k}
0	100,000	0.03
1	200,000	0.05
2	300,000	0.07

(ii) $i = 0.06$

Calculate the net premium reserve at the end of the first policy year.

(A) 7567

(B) 7703

(C) 7848

(D) 7978

(E) 8129

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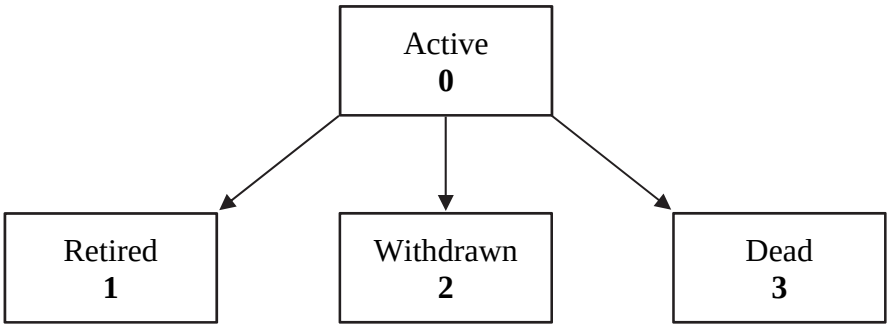
4. A pension plan models exits from employment using the following triple decrement model:



You are given the following additional information:

(i) There are 3000 active lives age 60.

(ii) At exact age 61, 20% of the active lives retire.

(iii) No retirements occur between integer ages.

(iv) $\mu_x^{02} = 0.05$ for $60 \le x < 62$

(v) $\mu_x^{03} = 0.02$ for $60 \le x < 62$

Calculate the number of individuals expected to withdraw between age 61 and age 62.

(A) 92

(B) 96

(C) 100

(D) 104

(E) 108

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5. You are given that the values of the Nelson-Aalen estimators at y_4 and y_5 are $\hat{H}_n(y_4) = 0.29727,$ and $\hat{H}_n(y_5) = 0.57000$. You are also given:

(i) There was 1 death at time y_4 .

(ii) There were 9 surrenders between times y_4 and y_5.

(iii) There were 3 deaths at time y_5.

(iv) Death and surrender are the only decrements.

(v) There were no new entrants to the study between times y_4 and y_5.

Calculate the number of people in the risk set at time y_4.

(A) 17

(B) 19

(C) 21

(D) 23

(E) 25

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6. A mortality improvement model similar to MP2014 uses an age-based cubic spline to interpolate between the base mortality rates, at time $t = 0$, and the long term rates applying from $t = 3$.

You are given the mortality improvement factors and their derivatives at the two end points:

$$\varphi(40,0) = 0.035, \qquad \varphi'(40,0) = 0.0$$
$$\varphi(40,3) = 0.005, \qquad \varphi'(40,3) = 0.0$$

Calculate the value of the spline for age 40 at $t = 2$.

(A) 0.009

(B) 0.011

(C) 0.013

(D) 0.015

(E) 0.017

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7. For a couple currently age x and y, you are given:

(i) The two lives have dependent future lifetimes.

(ii) $p_{x+k} = 0.80; \quad p_{y+k} = 0.90; \quad p_{x+k:y+k} = 0.77, \quad \text{for } k = 0, 1$

Calculate the probability that (x) and (y) both survive one year, and in the second year (x) dies and (y) survives.

(A) 0.100

(B) 0.114

(C) 0.130

(D) 0.139

(E) 0.151

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8. For a fully discrete whole life insurance of 100,000 issued to a select life age 68, you are given the following reserve assumptions:

(i) Mortality follows a select and ultimate mortality model with a 2-year select period.

(ii) Ultimate mortality rates follow the Standard Ultimate Life Table.

(iii) $i = 0.05$

(iv) $\ddot{a}_{[68]+1} = 12.3355$

Calculate the Full Preliminary Term Reserve at the end of the second policy year.

(A) 2516

(B) 2551

(C) 2589

(D) 2618

(E) 2653

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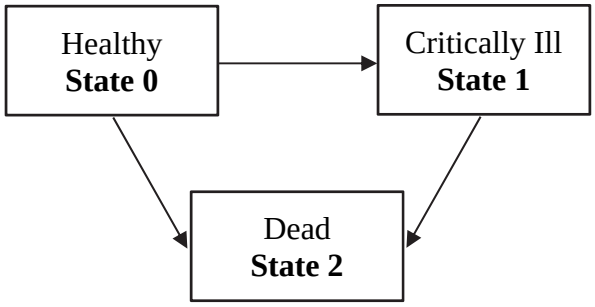
9. Bernard, (80), who is healthy, wants to purchase a critical illness insurance policy. The insurer uses the following model to value the benefits:



The insurance pays the following benefits:

- 50,000 at the year end if Bernard is critically ill at that time, having been healthy at the start of the year.
- 50,000 at the year end if Bernard dies during the year, having been critically ill at the start of the year.
- 100,000 at the year end if Bernard dies during the year, having been healthy at the start of the year.

You are given:

(i) $p_{80}^{00} = p_{81}^{00} = 0.90; \qquad p_{80}^{01} = p_{81}^{01} = 0.04$

(ii) $p_{81}^{12} = 1.00$

(iii) $i = 0.05$

Calculate the expected present value of the benefits payable during the first two years.

(A) 15,964

(B) 16,274

(C) 16,639

(D) 17,200

(E) 18,123

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10. For a 20-year deferred annuity-due on (65) you are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Deaths are uniformly distributed over each year of age.

(iii) The annuity pays 25,000 every six months starting at age 85.

(iv) Level monthly premiums are payable for 10 years.

(v) $i = 0.05$

Calculate the net monthly premium for this policy.

(A) 830

(B) 850

(C) 870

(D) 890

(E) 910

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11. A special 10-year deferred, fully discrete 20-year insurance is issued to (50). A benefit of 2,000,000 is paid on death between ages 60 and 80, and a benefit of 500,000 is paid on survival to age 80. There is no benefit on death before age 60.

You are given:

(i) Premiums are paid annually during the 10-year deferral period.

(ii) Commissions are 5% of premium.

(iii) Expenses of 260 per year are incurred at the start of each year following the deferral period.

(iv) Mortality follows the Standard Ultimate Life Table.

(v) $i = 0.05$

Calculate the annual gross premium using the equivalence principle.

(A) 29,800

(B) 30,000

(C) 30,200

(D) 30,400

(E) 30,600

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12. A two-year annuity-due on (98) pays 1000 every six months.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Between integer ages, two-thirds of deaths occur during the last six months of the year.

(iii) $i^{(2)} = 0.10$

Calculate the actuarial present value of the annuity-due.

(A) 2930

(B) 3010

(C) 3090

(D) 3170

(E) 3250

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13. A special 10-year insurance policy is issued to a healthy life age 55. The policy pays a benefit of 50,000 per year continuously during periods of sickness. Level net premiums of P per year are payable continuously while the policyholder is healthy.

The premium is calculated using the following assumptions:

(i) Sickness and mortality follow the Standard Sickness-Death model.

(ii) $i = 0.05$

Calculate P.

(A) 2544

(B) 3044

(C) 3544

(D) 4044

(E) 4544

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14. For a fully discrete whole life insurance of 1000 issued to (60), you are given the following information:

(i) Renewal expenses are 5 per year.

(ii) Commissions are 10% of the gross premium.

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) The valuation interest rate is 3% per year for the first 10 years, and 5% per year thereafter.

(v) The gross annual premium is 36.

Calculate the gross premium reserve at the end of the 9^{th} year.

(A) 71

(B) 73

(C) 75

(D) 77

(E) 79

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15. You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Deaths are uniformly distributed between integer ages.

(iii) $i = 0.04$

(iv) $A_{80}^{(4)} = 0.66227$

Calculate $A_{80.25}^{(4)}$.

(A) 0.6661

(B) 0.6670

(C) 0.6680

(D) 0.6692

(E) 0.6702

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16. An insurer issues fully discrete whole life policies of 100,000 to a group of lives age (60). The annual premium for each policy is 3765.

At the end of year 5, there were 100 policies still in force. In the following year:

(i) One policyholder died;

(ii) Expenses of 11% of each premium paid were incurred;

(iii) Actual interest earned was 5%.

You are also given that reserves per policy at the end of the 5^{th} and 6^{th} years are ${}_5V = 11{,}190$ and ${}_6V = 13{,}529$, respectively.

Calculate the profit on this group of policies in the 6^{th} year.

(A) 86,417

(B) 87,418

(C) 88,419

(D) 89,420

(E) 90,421

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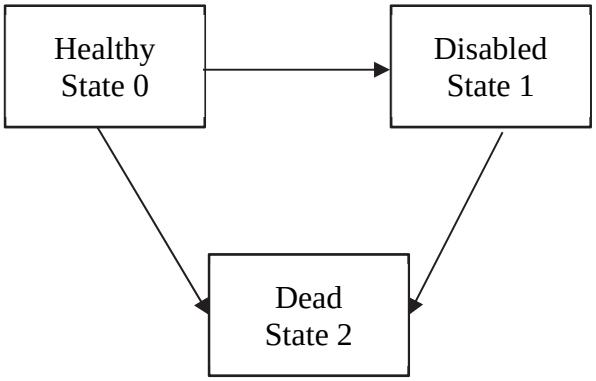
17. A disability income policy is issued to (x). The insurer uses the following model to value the benefits.



The benefit under the contract is a payment of 30,000 at the end of each year if (x) is Disabled at that time.

You are given the following information for a profit test of the contract:

(i) Premiums of 4000 are payable at the start of each year that (x) is Healthy.

(ii) Expenses are 2% of premiums.

(iii) ${}_1V^{(0)} = 2500; \quad {}_2V^{(0)} = 5100; \quad {}_1V^{(1)} = 54,000; \quad {}_2V^{(1)} = 52,100$

(iv) $p_{x+1}^{00} = 0.965; \quad p_{x+1}^{01} = 0.02$

(v) The insurer earns 8% interest on investments over the year.

Calculate $\text{Pr}_2^{(0)}$, the profit emerging at the end of the second year given that the policyholder is Healthy at the start of that year.

(A) 350

(B) 370

(C) 390

(D) 410

(E) 430

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18. Tom, who is age 45, is a member of a career average earnings pension plan. The benefit is a monthly annuity-due payable from age 65.

You are given the following valuation information:

(i) Tom's total pensionable earnings since entering the plan are 650,000.

(ii) Tom's salary for the year following the valuation date will be 90,000.

(iii) The accrual rate is 2% per year of service.

(iv) There are no exits before age 65 other than death.

(v) Mortality follows the Standard Ultimate Life Table.

(vi) Contributions are determined using the Traditional Unit Credit funding method.

(vii) $i = 0.05$

(viii) $\ddot{a}_{65}^{(12)} = 13.0870$

Calculate the normal contribution for Tom's retirement benefit.

(A) 7,235

(B) 7,546

(C) 7,857

(D) 8,168

(E) 8,479

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19. Kirsten, who is age 45, has just joined a final salary defined benefit pension plan. Final average salary is defined as the average salary in the two years before retirement.

The accrual rate is 1.6% per year of service.

You are given that salary increases follow the salary scale below.

x	s_x
45	2.539
46	2.637
47	2.730
48	2.816
49	2.897
⋮	⋮
58	3.382
59	3.432
60	3.454

Calculate Kirsten's projected replacement rate on retirement at age 60.

(A) 23.3%

(B) 23.8%

(C) 24.3%

(D) 24.8%

(E) 25.3%

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20. An insurer issues a 25-year disability income insurance policy to (55). Premiums of 9,600 per year are payable continuously while the policyholder is healthy (State 0), and a benefit of 50,000 is payable continuously while sick (State 1). No benefit is paid on death (State 2).

You are given:

(i) $\mu_{70}^{10} = 0.00087; \quad \mu_{70}^{12} = 0.04754$

(ii) ${}_{15}V^{(1)} = 300,860; \quad {}_{15}V^{(0)} = 40,942$

(iii) $i = 0.05$

Calculate $\frac{d}{dt}{}_tV^{(1)}$ at $t = 15$.

(A) –20,790

(B) –21,790

(C) –22,790

(D) –23,790

(E) –24,790

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Exam LTAM

SECTION B – Written-Answer

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1. (*10 points*) A fully discrete, joint life 10-year endowment insurance policy, with sum insured 100,000, is issued to two independent lives, both aged 70. Premiums are payable annually until the earlier of the first death or the contract maturity.

You are given:

(i) Acquisition expenses are 400 plus 50% of the first year's premium.

(ii) Renewal expenses are 2% of each subsequent premium.

(iii) Mortality follows Standard Ultimate Life Table.

(iv) $i = 0.05$

(v) The gross premium is $G = 10{,}658$.

(vi) L_0 is the gross loss at issue random variable.

(a) (*1 point*) Calculate the probability that both lives survive to the end of the contract.

(b) (*2 points*) Show that the expected loss at issue for the policy is $-4{,}470$ to the nearest 10. You should calculate the value to the nearest 1.

(c) (*3 points*)

(i) Show that ${}^2A_{70:70:\overline{10}|} = 0.44$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(ii) Show that the standard deviation of L_0 is 30,000 to the nearest 10,000. You should calculate the value to the nearest 10.

(d) (*2 points*) Now consider a portfolio of 100 identical, independent policies issued to couples both aged 70, with independent future lifetimes. Calculate the probability that the aggregate loss at issue is positive, using the Normal approximation to the aggregate loss distribution.

1. Continued

(e) (*2 points*) In a sub-portfolio of 10 identical, independent policies issued to couples with lives both age 70, the age at death of the 10 couples' lives turned out to be the following:

Policy Number	1	2	3	4	5	6	7	8	9	10
Age at death of first to die	75.2	78.5	80.5	82.1	83.5	83.8	83.9	91.3	91.5	95.8
Age at death of second to die	77.2	93.1	81.7	90.1	92.3	95.0	96.3	91.7	94.1	100.9

Without further calculation, state whether the value of the total profit realized in this sub-portfolio will be greater or less than the expected value of the profit for the sub-portfolio. Justify your answer.

2. (*10 points*) For a fully discrete three year term insurance of 100,000 on Anne, age 40, you are given:

(i) Anne's future mortality follows a double decrement model, where:

- Decrement 1 is death from Disease 1
- Decrement 2 is death from any cause except Disease 1

(ii) $\mu_{40+t}^{(1)} = A + Bc^{40+t}$ for all $t \geq 0$, where $A = 0.0001$, $B = 1.075 \times 10^{-5}$, $c = 1.12$

(iii) $\mu_{40+t}^{(2)} = 3\mu_{40+t}^{(1)}$, for all $t \geq 0$.

(iv) $i = 0.05$

(a) (*2 points*) Show that $p_{40}^{(\tau)} = 0.995$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

You are also given that ${}_2p_{40}^{(\tau)} = 0.99027$, and ${}_3p_{40}^{(\tau)} = 0.98462$.

(b) (*2 points*)

(i) Show that the expected present value at issue of Anne's death benefits is 1400 to the nearest 100. You should calculate the value to the nearest 1.

(ii) Show that the net annual premium for Anne's insurance is 490 to the nearest 10. You should calculate the premium to the nearest 0.1.

Anne purchases a policy rider that pays an additional 50,000 at the end of the year of death, if death is due to Disease 1.

(c) (*4 points*)

(i) Write down an integral expression for ${}_tq_{40}^{(1)}$, in terms of the dependent survival probabilities, ${}_sp_{40}^{(\tau)}$, and the force of mortality from Disease 1, $\mu_{40+s}^{(1)}$.

(ii) Using your expression from part (c) (i), prove that ${}_tq_{40}^{(1)} = \frac{1}{4}\,{}_tq_{40}^{(\tau)}$.

(iii) Show that the net premium for the Disease 1 rider is 60 to the nearest 10. You should calculate the premium to the nearest 0.1.

(d) (*2 points*) Calculate the net premium reserve at time 1 for the policy, including the Disease 1 rider.

3. (*9 points*) Let:

Y_1 denote the present value of an *n*-year term continuous annuity of 1 per year, issued to (*x*),

Y_2 denote the present value of a whole life continuous annuity of 1 per year, issued to (*x*),

Z_1 denote the present value of a continuous *n*-year endowment insurance of 1 issued to (*x*),

Z_2 denote the present value of a continuous whole life insurance of 1 issued to (*x*).

(a) (*2 points*)

(i) Write down an expression for Y_1 in terms of the force of interest, δ, and Z_1.

(ii) Write down an expression for Y_2 in terms of δ and Z_2.

(b) (*4 points*)

(i) Write down an integral expression for $\mathrm{E}[Z_1 Z_2]$, in terms of the probability density function (p.d.f.) of future lifetime, and interest rate functions.

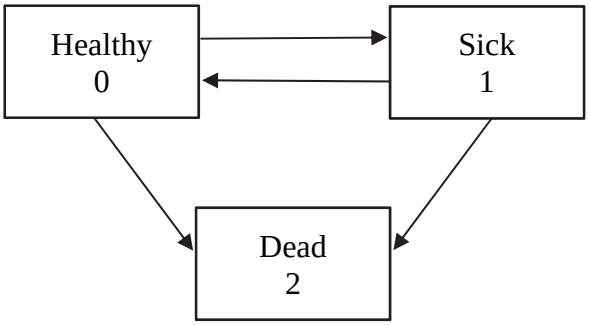
(Hint: use separate integrals for the period up to time *n*, and the period after time *n*).

(ii) Show that $E[Z_1 Z_2] = {}^2\bar{A}_{x:\overline{n}|} - v^n {}_nE_x \left(1 - \bar{A}_{x+n}\right)$

(c) (*2 points*) Using the expression from (b)(ii), derive an expression for $Cov(Y_1, Y_2)$.

(d) (*1 point*) State with reasons whether $Cov(Y_1, Y_2)$ will be positive, negative, or zero.

4. (*9 points*) You are using the following model to price a 10-year Disability Income Insurance policy issued to (x).



The product has the following features.

(i) Policyholders are healthy at the issue date.

(ii) A benefit of 20,000 is payable immediately on death within 10 years.

(iii) A continuous benefit of 1000 per year is paid while the policyholder is in the Sick state, during the 10-year term.

(iv) Premiums are payable continuously while the life is in the Healthy state.

(v) Maintenance expenses are paid at a continuous rate of 60 per year while the insured is alive.

(vi) Commissions are payable continuously at a rate of 5% of gross premium.

(vii) No benefits, premiums or expenses are payable beyond 10 years.

You are also given:

(viii) $\mu_{x+t}^{01} = 0.03; \quad \mu_{x+t}^{02} = 0.02; \quad \mu_{x+t}^{10} = 0.01; \quad \mu_{x+t}^{12} = 0.05, \quad \text{for } t > 0$

(ix) $\delta = 0.07.$

(x) $\bar{a}_{x:\overline{10}|}^{00} = 5.844; \quad \bar{a}_{x:\overline{10}|}^{01} = 0.684; \quad \bar{A}_{x:\overline{10}|}^{01} = 0.175; \quad \bar{A}_{x:\overline{10}|}^{02} = 0.151.$

(a) (*2 points*) Show that the gross premium rate, calculated using the equivalence principle, is 738 to the nearest 1. You should calculate the value to the nearest 0.1.

4. Continued

(b) (*2 points*) Calculate the probability that the life dies without becoming sick, within the 10-year term.

The insurer offers a new product with the following features:

- For any period of sickness that starts within the 10-year term, the benefit of 1000 per year is paid until death or recovery, even if that occurs after the 10-year term.
- There is an elimination period of 1 year.
- All other policy features and assumptions are the same as given above.

(c) (*1 point*) Define the term **elimination period** in the context of a disability income insurance product.

(d) (*2 points*) Show that the expected present value of a 1-year continuous sojourn annuity in State 1 is $\bar{a}^{\overline{11}}_{x:\overline{1}|} = 0.94$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(e) (*2 points*) Calculate the premium for this new policy, using the model and parameters given above.

5. (*10 points*) A life insurer issues a single premium whole life annuity-immediate, payable annually, with a 10-year guarantee, to Brad who is age 65.

The basis for calculating the premium and reserves is as follows:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

(iii) Commission is 2000 at issue.

(iv) Other acquisition expenses are 500.

(v) Maintenance expenses of 25 per year are payable with the annuity payments.

(vi) The premium is determined using the equivalence principle.

The single premium is 100,000.

(a) (*2 points*) Show that Brad's annuity payment is 7550 to the nearest 10. You should calculate the payment amount to the nearest 0.1.

The insurer conducts a profit test of this contract using the following three states:

State 0: Payments in progress; annuitant alive
State 1: Payments in progress; annuitant dead
State 2: Payments ceased; annuitant dead

You are also given the following assumptions for a profit test of the contract.

- Interest on insurer's funds: 6%
- All other assumptions follow the premium and reserve basis.

5. Continued

(b) (*1 point*) Write down values for the following probabilities:

(i) p_{73}^{01}

(ii) p_{73}^{02}

(iii) p_{73}^{12}

(c) (*2 points*) Show that the reserve at time 8 for a policy in State 0 is ${}_8V^{(0)} = 76{,}100$ to the nearest 100. You should calculate the value to the nearest 1.

(d) (*1 point*) Show that the reserve at time 8 for a policy in State 1 is ${}_8V^{(1)} = 14{,}100$ to the nearest 100. You should calculate the value to the nearest 1.

(e) (*4 points*)

(i) Show that $\text{Pr}_9^{(0)} = 760$ to the nearest 10. You should calculate the value to the nearest 0.1.

(ii) Show that $\text{Pr}_9^{(1)} = 140$ to the nearest 10. You should calculate the value to the nearest 0.1.

(iii) Calculate Π_9.

6. (*8 points*) Xiaobai is a member of a defined benefit pension plan which offers an annual pension of 2% of the final one year's salary for each year of service. The benefit is a whole life annuity-due paid monthly. At the valuation date, 1/1/2020, Xiaobai is 40 years old with 20 years' service. Xiaobai's salary during the 12 months prior to the valuation was 110,000.

You are given:

(i) All members in service at age 65 retire immediately.

(ii) There are no other retirements or withdrawals (other than death).

(iii) Mortality follows the Standard Ultimate Life Table.

(iv) $i = 0.05$

(v) Salary increases occur annually on 1 January each year.

(vi) Salaries are assumed to increase by 3% each year up to and including age 54, and by 2.5% each year from age 55.

(vii) Woolhouse's 2-term formula is used for monthly annuity functions.

(viii) The benefits are funded using the Projected Unit Credit method.

(a) (*3 points*)

(i) Show that the Actuarial Liability for Xiaobai's retirement benefits is 323,000 to the nearest 1,000. You should calculate the value to the nearest 1.

(ii) Calculate the Normal Contribution rate for Xiaobai's retirement benefits.

6. Continued

The plan proposes the following enhancement:

- A reversionary annuity benefit to the member's partner at 50% of the member's pension benefit, or
- For members who do not have a partner at their retirement dates, the pension would be increased by 5%.

Assume:

(i) 90% of members have partners at their retirement dates.

(ii) Partners are the same age as the member, for members with partners.

(iii) The mortality of members and their partners follows the Standard Ultimate Life Table.

(iv) Members and their partners have independent future lifetimes.

(b) (*4 points*)

(i) Calculate the revised Actuarial Liability for Xiaobai's enhanced retirement benefits (including the partner benefit).

(ii) Calculate the revised Normal Contribution rate for Xiaobai's enhanced retirement benefits.

(c) (*1 point*) The plan actuary is concerned that couples may have future lifetimes which are positively correlated. State with reasons whether the expected present value of the reversionary partner benefits would increase, decrease, or stay the same, in this case.

****END OF EXAMINATION****

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